$$\mathcal{L}\{(2-3t)^2 + e^{2t}(t-4) - 6\sin 3t\}$$

$$= \mathcal{L}\{4-12t+9t^2+te^{2t}-4e^{2t}-6\sin 3t\}$$

$$= 4\cdot\frac{1}{s} - 12\cdot\frac{1}{s^2} + 9\cdot\frac{2!}{s^3} + \frac{1}{(s-2)^2} - 4\cdot\frac{1}{s-2} - 6\cdot\frac{3}{s^2+9}$$

$$\mathcal{L}\{te^{2t}\} = -\frac{d}{ds}\mathcal{L}\{e^{2t}\}$$

$$= \frac{4}{s} - \frac{12}{s^2} + \frac{18}{s^3} + \frac{1}{(s-2)^2} - \frac{4}{s-2} - \frac{18}{s^2+9}$$

$$= -\frac{d}{ds}\frac{1}{s-2}$$

$$= \frac{1}{(s-2)^2}$$

IF $f(t) = \begin{cases} 0, & 0<t<2 \\ t, & t>2 \end{cases}$, FIND $\mathcal{L}\{f\}$

$$\begin{array}{cc} u & dv \\ t & +e^{-st} \\ 1 & -\frac{1}{s}e^{-st} \\ 0 & \frac{1}{s^2}e^{-st} \end{array}$$

$$\int_0^\infty e^{-st}f(t)\,dt = \cancel{\int_0^\infty e^{-st}t\,dt}$$

$$= \int_0^2 e^{-st}\cdot 0\,dt + \int_2^\infty e^{-st}\cdot t\,dt$$

$$= 0 + \lim_{N\to\infty}\int_2^N te^{-st}\,dt$$

$$= 0 + \lim_{N\to\infty}\left(-\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\right)\Big|_2^N$$

$$= \lim_{N\to\infty}\left(-\frac{1}{s}(Ne^{-sN} - 2e^{-2s}) - \frac{1}{s^2}(e^{-sN} - e^{-2s})\right)$$

$$= -\frac{1}{s}(0-2e^{-2s}) - \frac{1}{s^2}(-e^{-2s})$$

$$= e^{-2s}\left(\frac{2}{s}+\frac{1}{s^2}\right) = \frac{(2s+1)e^{-2s}}{s^2}$$

$\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$ IF $f \ldots$

$$\mathcal{L}\{f''\} = \mathcal{L}\{(f')'\} = s\mathcal{L}\{f'\} - f'(0)$$
$$= s(s\mathcal{L}\{f\} - f(0)) - f'(0)$$
$$= s^2\mathcal{L}\{f\} - sf(0) - f'(0)$$

$$\mathcal{L}\{f'''\} = \mathcal{L}\{(f'')'\} = s\mathcal{L}\{f''\} - f''(0)$$
$$= s(s^2\mathcal{L}\{f\} - sf(0) - f'(0)) - f''(0)$$
$$= s^3\mathcal{L}\{f\} - s^2f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}\} = s^n\mathcal{L}\{f\} - s^{n-1}f^{(0)}(0) - s^{n-2}f^{(1)}(0) - s^{n-3}f^{(2)}(0) - \cdots - f^{(n-1)}(0)$$

$$\boxed{\mathcal{L}\{f^{(n)}\} = s^n\mathcal{L}\{f\} - \sum_{k=1}^{n} s^{n-k}f^{(k-1)}(0)}$$

SOLVE $y'+y=1,\ y(0)=3$

$$\mathcal{L}\{y'+y\}=\mathcal{L}\{1\}$$

$$\mathcal{L}\{y'\}+\mathcal{L}\{y\}=\frac{1}{s} \qquad Y=\mathcal{L}\{y\}$$

$$sY-y(0)+Y=\frac{1}{s}$$

$$(s+1)Y=\frac{1}{s}+3$$

$$Y=\frac{\frac{1}{s}+3}{s+1}\frac{s}{s}=\mathcal{L}\{y\}=\left(\frac{1+3s}{s(s+1)}\right)^{*s(s+1)}=\left(\frac{\cancel{A}^{\,1}}{s}+\frac{\cancel{B}^{\,2}}{s+1}\right)^{*s(s+1)}$$

$$1+3s=A(s+1)+Bs$$

$s=0$: $1=A$

$s=-1$: $-2=-B \rightarrow B=2$

MANDATORY SANITY CHECK: $s=-2$ $\quad \frac{1-6}{(-2)(-1)}=\frac{-5}{2}$

NOT $0,\pm1$
NOT ANYTHING
ALREADY USED

$$\frac{1}{-2}+\frac{2}{-1}=-\frac{1}{2}-2=-\frac{5}{2}$$

=

✓

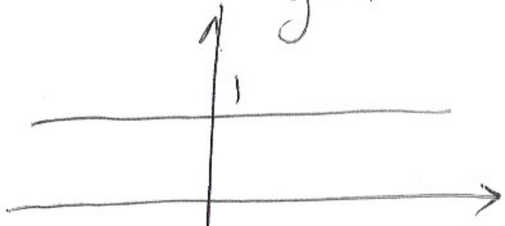
$$\mathcal{L}\{y\}=\frac{1}{s}+\frac{2}{s+1}$$

$$y=1+2e^{-t}$$

IF $\mathcal{L}\{y\} = Y$ WE SAY $\mathcal{L}^{-1}\{Y\} = y$

+ y IS CONT ON $[0,\infty)$

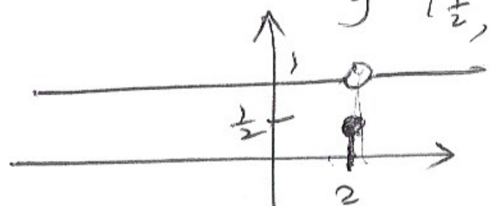
$y=1$



$\mathcal{L}\{y\} = \frac{1}{s}$

$= \int_0^\infty e^{-st}\cdot 1\,dt$

$y = \begin{cases} 1, & \text{IF } x \neq 2 \\ \frac{1}{2}, & \text{IF } x = 2 \end{cases}$



$\mathcal{L}\{y\} = \frac{1}{s}$

$= \int_0^\infty e^{-st}\cdot 1\,dt$

$$\mathcal{L}^{-1}\left\{\frac{2}{s^5}\right\} = \frac{1}{12}t^4$$

$$\frac{2}{4!}\mathcal{L}\{t^4\} = \frac{4!}{s^{4+1}} \cdot \frac{2}{4!} = \frac{2}{s^5}$$

$$\mathcal{L}\left\{\frac{1}{12}t^4\right\} = \frac{2}{s^5}$$